It is now assumed that the upstream and downstream states of the flow differ only by small quantities which will be denoted by tildas

$$\rho_2 = \rho_1(1 + \tilde{\rho}), \quad u_2 = u_1(1 + \tilde{u}), \quad v_2 = u_1\tilde{v}$$

where it has been assumed that the initial flow direction lies along the x axis. Elimination of the shock direction between Eqs. (10) and (12) gives

$$(\tilde{v} + \tilde{\rho}\tilde{v})\tilde{v} + (\tilde{\rho} + \tilde{u} + \tilde{\rho}\tilde{u})\tilde{u} = 0 \tag{13}$$

Now expand the energy Eq. (11) to second order using the isentropic relation to eliminate the pressure. Then taking $M_1 = 1$ in the second-order terms gives

$$\tilde{\rho} + M_1^2\tilde{u} + [\gamma - 1)/2]u^2 + \tfrac{1}{2}\tilde{v}^2 = 0 \tag{14}$$

By means of this form of the energy equation, the terms of the second bracket of Eq. (13) are seen to be of order $(1 - M_1^2)\tilde{u}^2$ and $\tilde{u}^3$. Consequently in transonic flow $\tilde{v}^2$ is negligible compared to $\tilde{u}^2$ and can be dropped from Eq. (14). Similarly $\tilde{\rho}\tilde{v}^2$ is negligible in Eq. (13), which after $\tilde{\rho}$ is eliminated reads

$$\tilde{v}^2 + (1 - M_1^2)\tilde{u}^2 - [(\gamma + 1)/2]\tilde{u}^3 = 0 \tag{15}$$

This equation is the usual transonic expansion of the exact shock polar

$$\tilde{v}^2 = \tilde{u}^2 \frac{(M_1^2 - 1) + [(\gamma + 1)/2]M_1^2\tilde{u}}{1 - [(\gamma + 1)/2]M_1^2\tilde{u}}$$

5. Conclusion

The equations of unsteady potential flow can be cast into conservation form for use in finite-difference applications. The conservation form provides the correct shock jump for shock smearing applications, and is accurate to third order in the shock strength.The most restrictive condition arises from the condition of tangential velocity continuity. In contrast to the results from the complete system of conservation laws, the simplified equations do not admit tangential discontinuities. This behavior must be expected from the imposition of the irrotationality condition throughout the finite difference mesh. Problems with strong tangential discontinuities must be treated by cutting the mesh system along the discontinuity and including it explicitly through boundary conditions. This restriction is not especially severe since important tangential discontinuities should be treated explicitly even with the full equations.[5] The artificial viscosity is usually so large that it diffuses the discontinuity very strongly within a few mesh intervals.

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The Effect of Angle of Attack on Boundary-Layer Transition on Cones

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Nomenclature

k = parameter related to circumferential gradient of circumferential velocity on the windward ray of a cone

$$k = \left(\frac{2}{3\sin\theta_c}\right)\left(\frac{1}{V_e}\frac{\partial w}{\partial \Phi}\right)_{\Phi} = 0°$$

M = Mach number
Re_θ = Reynolds number based on momentum thickness
s_t = length to transition along a cone generator
V = velocity along a streamline
w = circumferential component of velocity
α = angle of attack
θ_c = cone half-angle
μ = viscosity
ρ = density
Φ = angular coordinate around the cone ($\Phi = 0°$; windward ray)

Subscripts
e = boundary-layer edge condition
∞ = freestream condition

PREVIOUS transition investigations on cones at angle of attack[1–3] have been mainly devoted to the windward and leeward rays and to angles of attack less than the cone half-angle. Although the results of these studies show a fairly consistent qualitative behavior (i.e., transition moves forward on the leeward ray and aft on the windward), a quantitative, parametric description has not been found. In an effort to satisfy the need for more data and to provide a definition of the parameters affecting transition the present study was initiated. The objectives of the investigation were to 1) obtain a detailed map of transition around a cone of angle of attack and 2) attempt a correlation of the results with existing data.

Tests were conducted in the Ames Research Center's 3.5 ft hypersonic wind tunnel at a freestream Mach number of 7.4. The models were 5° and 15° half-angle cones at angles of attack from 0° to 20°. The wall and total temperatures were nominally 305°K and 833°K, respectively, and total pressures ranged from 2.165×10^6 to 1.209×10^7 N/m^2. Boundary-layer transition was determined from heating rate distributions (deduced from thermocouple histories) as described in Ref. 4.

Transition Reynolds numbers were defined using boundary-layer edge conditions calculated by the method of characteristics.[5] Edge conditions for angles of attack greater than the cone half-angle were obtained using the following approximations. 1) The 15° cone edge, conditions at $\alpha = 20°$ were obtained by extrapolation from $\alpha \leqq 15°$. 2) The windward ray conditions on the 5° cone for $\alpha > 5°$ were calculated using characteristics by replacing the leeside with an ellipse so that the leeward ray was aligned with the freestream velocity vector. 3) Conditions on the leeward ray of the 5° cone at $\alpha = 6°$ were obtained by extrapolation from $\alpha \leqq 5°$. In formulating the transition Reynolds number the velocity along the local streamline was used in conjunction with the distance measured from the apex along conical rays.

The effect of angle of attack on local transition Reynolds number is illustrated in Fig. 1. It is obvious from the figure that the influence of angle of attack on transition Reynolds number is a function of meridian angle, Φ. For example, on the windward ray of the 15° cone transition Reynolds numbers show an initial, slight increase with α and then a decrease; whereas, leeward ray Reynolds numbers decrease rapidly with α. On the leeward ray the effect of α on transition is similar for both models, however, the windward ray results are considerably different.

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Windward-ray transition Reynolds numbers increase monotonically with angle of attack on the 5° cone; however, on the 15° cone there is a net decrease in transition Reynolds numbers for α from 0° to 20°. At $\alpha = 20°$ the local transition Reynolds number is at least (arrow indicates transition off model) four times the $\alpha = 0°$ value on the 5° cone and only 60% of the $\alpha = 0°$ value on the 15° cone. It will be shown that the contrasting behavior of transition on the windward ray is related to differences in local conditions, cone angle, and crossflow velocity gradient.

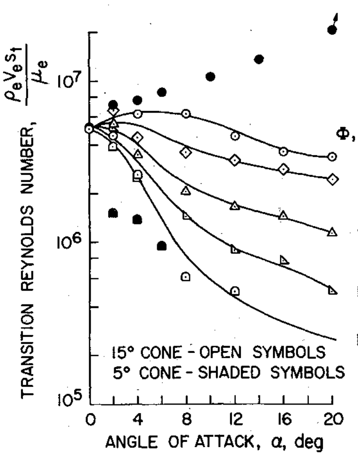

Fig. 1 Effect of angle of attack on local transition Reynolds number; $\theta_c = 5°$ and 15°, $M_\infty = 7.4$.

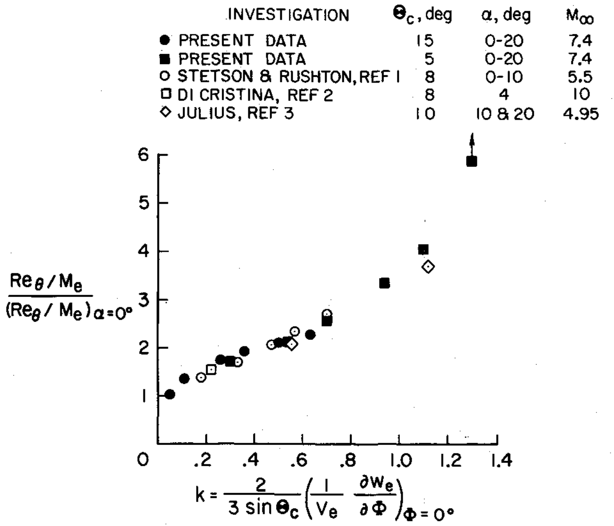

Fig. 2 Correlation of the beginning of transition on the windward ray of cones.

Transition on cones at angle of attack can, potentially, be affected by several parameters, such as crossflow velocity and velocity gradient, pressure gradient along streamlines, and changes in local Mach number. With so many variables to consider, it is desirable to eliminate some effects where possible so that the influence of one or two parameters can be isolated. The windward centerline affords such a possibility since there is no crossflow velocity or pressure gradient along the streamline, and the crossflow velocity gradient (derivative of the circumferential velocity in the circumferential direction) and local conditions can be adequately predicted.[5] Consequently, a correlation based on changes in local conditions and crossflow velocity gradient was attempted for transition data on the windward ray, using the following procedure.

Previous investigations on cones at $\alpha = 0°$ (e.g., Ref. 6) have shown that (at a constant freestream unit Reynolds number) the effect on transition of changes in local flow conditions can be accounted for by an approximately linear relationship between local-momentum-thickness Reynolds number, at transition, and local Mach number. In the present correlation it was assumed that a similar linear dependence exists at angle of attack and the deviations from the $\alpha = 0°$ relationship are a function of the crossflow velocity gradient parameter k of Ref. 7. The local conditions were calculated using the previously described characteristics solution, and the momentum thicknesses at transition were calculated by integrating the boundary-layer profiles tabulated in Ref. 7 and interpolating at the present test conditions. A satisfactory correlation of windward ray transition data on cones was achieved, as shown in Fig. 2.† In addition to the present data, those of Refs. 1–3 were also correlated. Selection of data from other investigations was contingent upon the beginning of the transition being defined in the same manner, i.e., from heat-transfer distributions. Also, all of the experiments were performed in air. The results indicate that the linear relationship between local-momentum thickness Reynolds number and edge Mach number still exists at angle of attack.

The extension of this correlation to the case of an arbitrary streamline is certainly an attractive possibility. In the general case, however, the velocity gradient may not be the correlating parameter. In this instance a parameter related to streamline spreading may be more appropriate. For example, for the specific case of the windward ray of a cone, Vaglio-Laurin[9] has shown that the variable k is related to streamline spreading.

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† This figure replaces Fig. 6 of Ref. 8.